# MATHEMATICAL SENTENCES

**Words to be Learned**

**Exercise 1**

*Read these international words and try to guess their meaning.*

equality [i(:)'kwɔlıtı] *n*, reflexive [rı'fleksıv] *a*, symmetric [sı'metrık] *a*, transitive ['trænz(s)ətıv] *a*, opposite ['ɔpəzıt] *a*, correct [kə'rekt] *a*, incorrect [**ֽ**ınk(ə)'rekt] *a*, essential [ı'sen∫(ə)l] *a*, inequality [**ֽ**ıni(:)'kwɔlıtı] *n*, false [fɔ:ls] *a*, express [ı(e)k'spres] *v*.

**Exercise 2**

*Repeat after the teacher.*

|  |  |
| --- | --- |
| decide [dı'saıd] решать  part [pα:t] *n* часть; *v* делить  relation [rı'leı∫(ə)n] отношение  relate [rı'leıt] относить (к)  satisfy ['sætısfaı] удовлетворять  even ['i:v(ə)n] даже  while [waıl] в то время как, между тем  feature ['fi:t∫ə] черта, свойство  involve [ın'vɔlv] включать в себя; влечь за собой  either ['aıðə] 1. любой (из); 2. либо  determine [dı'tɜ:mın] определять  direct [d(a)ı'rekt] *а* прямой; *v* направлять  concise [kən'saıs] сжатый  understanding [**ֽ**лndə'stændıŋ] понимание  understand (understood) понимать  merely ['mıəlı] просто, только | whether ['weðə] ли *(союз)*  across [ə'krɔs] через, сквозь  conventional [kən'ven∫(ə)n(ə)l] привычный, общепринятый  replace [rı'pleıs] замещать, заменять  choose [t∫u:z] (chose [t∫əuz], chosen ['t∫əuzən]) выбирать  wrong [rɔŋ] *а* ошибочный  be wrong ошибаться  member ['membə] член  constantly ['kɔnstəntlı] постоянно  respect [rıs'pekt] *n* 1. отношение; 2. уважение  draw [drɔ:] (drew [dru:], drawn [drɔ:n]) 1. чертить, рисовать; 2. тянуть, выводить  namely именно  include [ın'klu:d] включать |

**Notes**

1. you will remember – вы, по-видимому, помните
2. either true or false – либо истинное, либо ложное
3. both true and false – как истинное, так и ложное
4. to decide whether a closed sentence is – чтобы решить, является ли замкнутое выражение
5. we check to see – мы проверяем, чтобы убедиться
6. there is nothing incorrect about writing – вовсе не ошибочно написать
7. in other words – иными словами

**Exercise 3**

*Repeat after the speaker. Mind that one and the same sound can be expressed in writing by various letters and combinations of letters.*

[i:] – feed, see, seems, meet, need, beta, theme, he, mete, Pete, these, scene, even, secret, complete, meat, mean, heat, speak, field, piece;

[α:] – arm, far, article, card, father, rather, yard, artist, margin, mask, fast, last, pass, ask, answer, half;

[ɔ:] – form, glory, more, oral, short, tall, wall, small, walk, salt, cause, because, author, autumn, door, floor, saw, law, draw, war, warm, quarter, your, four, course, thought, brought, taught;

[ɜ:] – first, firm, circle, thirty, third, curve, hurt, urgent, turn, germ, serve, herd, nerve, person, earn, early, work, word, worth.

**Exercise 4**

*Write questions making use of the question words.*

1. Either of these two examples could show the real situation. (in what way) 2. You will have to replace one member of the equality with another. (why) 3. She gave a detailed explanation of their method of research. (when) 4. I expect a concise definition instead of this general talk. (from whom) 5. These conventional symbols are accepted by scientists. (where) 6. He did not want to be involved in this long and uninteresting discussion. (why) 7. The most essential part of the program is being considered now. (by whom) 8. They use both direct and indirect methods of investigation. (in what case) 9. This is the most correct and concise definition given. (by whom) 10. This statement is false and I can prove it. (how)

**Exercise 5**

*Read these words.*

due, good, union, knew, early, ought, worth, secret, draw, circle, fast, army, field, seem, beta, warm, urgent, ruler, tall, yard, mean, bought, salt, third, fourth, work, article, true, wall, earn, tune, group, curve, law, soon, watch, person, autumn, war, yours, drawing, author

**Exercise 6**

*Ask a general question about each sentence below.*

1. They included all the necessary data in their recent article. 2. The first element of the relation contains the letter *y.* 3. Certain characteristic features of the process are being considered at the moment. 4. The student placed the equality sign between the two parts of the equation. 5. The plus sign has to be replaced by the opposite sign. 6. The manager was asked direct questions in the course of the discussion. 7. They were satisfied with the results of the international conference. 8. There exist false statements in mathematics. 9. You will be able to apply either the first method or the second one. 10. More than fifteen people were involved in the experiment. 11. He is constantly arguing with his sister.

**Exercise 7**

*Disagree with the statements.*

Т.: Не finds his position false.

St.: He does not find his position false.

1. The members of their scientific group took part in the seminar. 2. He decided to agree to that appointment. 3. The given relation satisfies the symmetric axiom. 4. This article is concerned with the most essential political problems of today. 5. It was meaningless to use direct methods in such experiments. 6. You will be able to replace the old parts of the machine with the new ones. 7. He understands children better than she does. 8. This seminar is as popular with our students as Alexandrov’s seminar. 9. He chose one of the two possibilities. 10. He can draw very well.

**Exercise 8**

*Read the text below and find in it answers to the following questions.*

1. What is an equation? 2. Is it always incorrect to write a false mathematical sentence? 3. Which basic axiom does the relation *a* = *a* satisfy? 4. Does the sign of non-equality (≠) tell you which numeral names the greater or the lesser of the two numbers?

**Something about Mathematical Sentences**

In all branches of mathematics you need to write many sentences about numbers. For example, you may be asked to write an arithmetic sentence that includes two numerals which may name the same number or even different numbers. Suppose that for your sentence you choose the numerals 8 and 11–3 which name the same number. You can denote this by writing the following arithmetic sentence, which is true: 8=11–3.

Suppose that you choose the numerals 9+6 and 13 for your sentence. If you use the equal sign (=) between the numerals you will get the following sentence 9+6=13. But do 9+6 and 13 both name the same number? Is 9+6=13 a true sentence? Why or why not?

You will remember1 that the symbol of equality (=) in an arithmetic sentence is used to mean *is equal to*.Another symbol that is the symbol of non-equality (≠)is used to mean *is not equal to****.*** When an equal sign (=) is replaced by a non-equal sign (≠), the opposite meaning is implied. Thus the following sentence (9+6≠13) is read: nine plus six is not equal to thirteen. Is it a true sentence? Why or why not?

An important feature about a sentence involving numerals is that it is either true or false2, but not both.

A mathematical sentence that is either true or false, but not both is called a closed sentence. To decide whether a closed sentence3 containing an equal sign (=) is true or false, we check to see4 that both elements, or expressions, of the sentence name the same number. To decide whether a closed sentence containing a non-equal sign (≠)is true or false, we check to see that both elements do not name the, same number.

As a matter of fact, there is nothing incorrect or wrong, about writing5 a false sentence; in fact, in some mathematical proofs it is essential that you write false sentences. The important thing is that you must be able to determine whether arithmetic sentences are true or false.

The following properties of equality will help you to do so.

Reflexive: *a* = *a*

Symmetric: If *a* = *b*,then *a* = *b*.

Transitive: If *a* = *b* and *b* = *c*,then *a* = *c.*

The relation of equality between two numbers satisfies these basic axioms for the numbers *a*, *b*,and *c*.

You already know that drawing a short line across the = sign (equality sign) we change it to ≠sign (non-equality sign). The non-equality symbol (≠)implies either of the two things, namely: is greater than or is less than. In other words6, the sign of non-equality (≠)in 3+4≠6 merely tells us that the numerals 3+4 and 6 name different numbers; it does not tell us which numeral names the greater or the lesser of the two numbers.

If we are interested to know which of the two numerals is greater we use the conventional symbols meaning less than (<) or greater than (>). These are inequality symbols or ordering symbols because they indicate order of numbers. If you want to say that six is less than seven, you will write it in the following way: 6<7. If you want to show that twenty is greater than five, you will write 20>5.

The signs which express equality or inequality (=, ≠,>, <) are called relation symbols because they indicate how two expressions are related.

**Exercise 9**

**a)** *Shorten the text retaining the most essential information;*

**b)** *Be prepared to retell the text.*

**Exercise 10**

**a)** *Listen and repeat. Guess the meaning of the words in italics;*

**b)** *Arrange the words according to the parts of speech they belong to.*

part *n, v* – *partly*;relation – *interrelation*;satisfy – *satisfaction*;to involve *–involvement*; to direct – *directly* – *directness* – *indirect – indirectly – direct a*;false *a* –*falsify* – *falsification*;symmetric – *symmetry n*;concise *a* – *concisely*;to include – *inclusion*;constantly – *constant a*;to replace – *replacement*;to decide – *decision*;to determine – *determination*;opposite – *opposition* – *to oppose*;correct *a* – *correctly* – *to correct* – *correctness* – *incorrectly*; to express – *expressive.*

**Exercise 11**

*Write down your answers* (+, –) *to the following questions*.

1. Do you need to write sentences about numbers in all branches of mathematics? 2. Do the numerals 8 and 11–3 name the same number? 3. Do 9+6 and 13 both name the same number? 4. Is the sentence 9+6=13 a true sentence? 5. Can a sentence involving numerals be both true and false? 6. Is a mathematical sentence that .is either true or false, but not both, called a closed sentence? 7. Is everyday language more direct and concise than symbolic language? 8. Must we have a good understanding of the meaning of each symbol? 9. Does the symbol of non-equality (≠)imply that one member of the mathematical sentence is greater than the other? 10. Are relation or ordering symbols important?

**Exercise 12**

*Ask questions to which the following could be answers.*

1. There is a certain difference between an equation and an equality. 2. There exist true and false mathematical sentences. 3. Both members of the equation name the same number. 4. We are to decide whether this sentence is true. 5. It is necessary to know whether this sentence is correct. 6. This relation of equality satisfies the following axioms. 7. The given symbol ≠ means is not equal to. 8. The following symbol > means greater than. 9. This mathematical sign implies the opposite meaning. 10. In mathematics it is possible to write a false sentence. 11. Everyone who is concerned with mathematics must have a good understanding of the meaning of each symbol. 12. The symbol of non-equality ≠ implies either greater than or less than. 13. These are ordering symbols because they indicate the order of numbers.

**Exercise 13**

*Write in the negative.*

1. I decided to take part in this work. 2. He is an active member of their group. 3. I am satisfied with their reaction to our publication. 4. This feature is characteristic of him. 5. He is determined to provide us with all the information possible. 6. This work has to include direct methods of control. 7. The child can draw much better. 8. This process can be constantly reproduced in the lab. 9. These symbols are as conventional as the ones used in this book. 10. They have to study the relation between these two discoveries at present. 11. I agreed with him in just one respect. 12. I believe you will choose the right solution.

**Exercise 14**

**a)** *Read the text below and entitle it;*

**b)** *Write a few questions about the text in the form of a plan;*

**c)** *Be prepared to discuss the text.*

In this lesson the students learn to recognize, understand, and use the distributive [dıs'trıbjətıv] property of multiplication over addition. Further the students discover that the properties of operations are fundamental to understanding the structure of mathematics. They make the usual arithmetic manipulations secure.

Multiplication is distributive over addition in the set of natural numbers. That is, for any three natural numbers *a*, *b*,and *c*, *a(b+c)*=*ab+ac.* This property can be extended (можно расширить) to include more than two addends. Since the set of natural numbers is closed under addition, *a+b* names, a natural number (when *а* and *b* natural numbers). Therefore, the distributive property is extended as follows.

*a(b+c+d)*=*a*[*b+*(*c+d*)]=*ab+a(c+d)*=*ab+(ac+ad)*=*ab+ac+ad*

It is important that the students also recognize, by the property of equality, that the above statements are reversible ([rı'vɜ:səbl] обратимы). For example *ab+ac*=*a(b+c)* and *ab+ac+ad*=*a(b+c+d).*

Without the concept of symmetry of equality, the students sometimes find it difficult to understand the properties of operations. They find it easy to work from left (левый) to right (правый) but they meet difficulties while using the distributive property in a factoring situation, as given below.

5*n*+5*r*=5(*n+r*)

The use of the associative, commutative [ke'mju:tətıv], and distributive properties in working on equivalent expressions is extremely important in the development of mathematical thought ([θɔ:t] *зд*. мышление) and in the understanding of the structure of mathematics.

**Exercise 15**

*Read these words. Guess the meaning of the words in italics.*

to **ֽ**under'stand – **ֽ***under*'*standable*;dis'tributive – *to dis*'*tribute* – **ֽ***distri*'*bution*;structure – '*structural*;to in'clude – *in*'*cluston*; to close *– closed – closure – close a*;to re'verse *– re*'*versible*;'difficult– '*difficulty*; *ass*'*ociative – a***ֽ***ssoci*'*ation – to associate* [a'səu∫ıeıt], [a'səusı**ֽ**eıt]; ex'tremely – *extreme* [ıks'tri:m] *a* – *ex*'*tremist*; *thought* – *thoughtful* – *thoughtless*;de'termine –*de***ֽ***termi*'*nation*;'constant – '*constancy n*;di'rectly – *indi*'*rectly.*

**Exercise 16**

*Say the following in English.*

1. Уравнение содержит знак равенства? 2. Эти две части называют членами уравнения. 3. Математические выражения могут быть либо истинными, либо ложными. 4. Вам следует поставить знак неравенства между членами этого отношения. 5. Знак ≠заменяется знаком = в этом выражении. 6. Эти символы предполагают (заключают в себе) различные значения. 7. Они не смогли понять значения этого выражения. 8. Вы знаете общепринятые математические символы? 9. Это доказательство более правильно? 10. Язык математики есть самый прямой и сжатый язык.